$X_1, \dots, X_n$ — n r.v.'s

$a_1, \dots, a_n$ — n constants

$$\mathbb{E}\left(\sum_i a_i X_i\right) = \mathbb{E}\left(\underbrace{a_1 X_1}_{i=1} + \underbrace{a_2 X_2}_{i=2} + \dots + \underbrace{a_n X_n}_{i=n}\right)$$

Thm 3.11 $\quad \mathbb{E}\left(\sum_i a_i X_i\right) = \sum_i a_i \mathbb{E}(X_i)$

$$a_1 \mathbb{E}(X_1) + a_2 \mathbb{E}(X_2) + \dots + a_n \mathbb{E}(X_n)$$

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

If X + Y are independent, $Var(X+Y) = Var(X) + Var(Y)$

$Var(X+Y+Z) = ?$

applied

$$\begin{aligned}
Var(X+Y+Z) &= Var((X+Y)+Z) \\
&= Var(X+Y) + Var(Z) + 2Cov(X+Y, Z) \\
&= Var(X) + Var(Y) + 2Cov(X,Y) + Var(Z) \\
&\quad + 2\boxed{Cov(X+Y, Z)}
\end{aligned}$$

def of cov / alt form of cov.

$$\begin{aligned}
\boxed{Cov(X+Y, Z)} &= \mathbb{E}((X+Y)Z) - \mathbb{E}(X+Y)\,\mathbb{E}(Z) \\
&= \mathbb{E}(XZ + YZ) - (\mathbb{E}(X) + \mathbb{E}(Y))\,\mathbb{E}(Z) \\
&= \mathbb{E}(XZ) + \mathbb{E}(YZ) - \mathbb{E}(X)\mathbb{E}(Z) - \mathbb{E}(Y)\mathbb{E}(Z) \\
&= (\mathbb{E}(XZ) - \mathbb{E}(X)\mathbb{E}(Z)) + (\mathbb{E}(YZ) - \mathbb{E}(Y)\mathbb{E}(Z)) \\
&\overset{\text{def (alt form of cov)}}{=} Cov(X,Z) + Cov(Y,Z)
\end{aligned}$$

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

$$Var(X+Y+Z) = Var(X) + Var(Y) + Var(Z) + 2Cov(X,Y) + 2Cov(X,Z) + 2Cov(Y,Z)$$

$$Var(X+Y+Z+W) =$$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

X_1 returns on asset 1

X_2 returns on asset 2 } r.v.'s

$\mathbb{E}(w_1 X_1 + w_2 X_2) =$ ___

$Var(w_1 X_1 + w_2 X_2)$

rbinom(10, 1, 0.5)

? ? ? ? ? ? ? ? ?

0,1

$X_1, X_2, X_3, \dots, X_{10}$

Sample average

$$\bar{X}_n = \frac{X_1 + X_2 + X_3 + \dots + X_{10}}{10}$$

dist r.v. →

$$\bar{X}_n = \frac{X_1 + X_2 + \dots + X_n}{n}$$

n = sample size

~~$P(X=2) = \frac{1}{10}$, $P(X=3) = \frac{1}{10}$, $P(X=5) = \frac{8}{10}$~~

$\mu = \mathbb{E}(X) = 4.5$ $\sigma^2 = Var(X) = 1.05$

$\mathbb{E}(\bar{X}_n) = \mu$ ← not observable in real world

could be observed

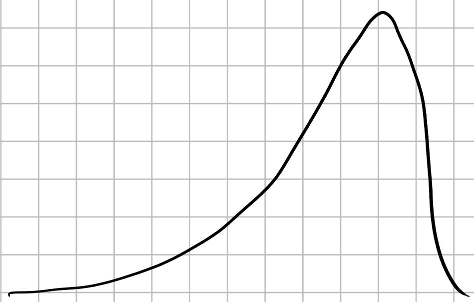
could be computed

If $\mathbb{E}(\bar{X}_n) \neq \mu \Rightarrow$ $\mathbb{E}(\bar{X}_n) > \mu$ or $\mathbb{E}(\bar{X}_n) < \mu$

$\bar{X}_n$ overestimates μ

$\bar{X}_n$ underestimates μ

trimmed mean



n = 10

n = 100

$Var(\bar{X}_n) = \frac{\sigma^2}{n}$

$Var(X) = E(X - E(X))^2$

$Var(\bar{X}_n) = E(\bar{X}_n - E(\bar{X}_n))^2$

μ

n = 10

$Var(\bar{X}_n) = \frac{1.05}{10} = 0.105$

n = 100

$Var(\bar{X}_n) = \frac{1.05}{100} = 0.0105$