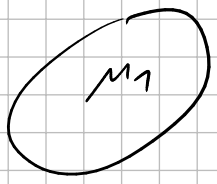
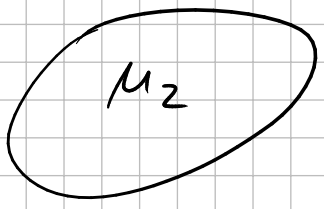
Lecture 5d

$$\mathbb{P}\left(\bar{X}_n - 2\sigma/\sqrt{n} \le \mu \le \bar{X}_n + 2\sigma/\sqrt{n}\right) \ge 3/4$$





$\mu_1 - \mu_2$

pop^n 1 pop^n 2

$H_0: \mu_1 = \mu_2$

$\mu = 4.5 \quad \sigma^2 = 1.05$

$X_1, \dots, X_{10}$ IID from $\mathbb{P}(X=2) = \frac{1}{10}, \mathbb{P}(X=3) = \frac{1}{10}, \mathbb{P}(X=5) = \frac{8}{10}$

Find $\mathbb{P}(\bar{X}_{10} \ge 4.2)$.

X_1	X_2	X_3	...	X_{10}	$\bar{X}_{10}$	prob.
5	5	-	...	5	5	$(\frac{8}{10})^{10}$
2	2	5	5...	5	4.4	$(\frac{1}{10})^2(\frac{8}{10})^8$
2	2	...		2	2	

Central limit theorem

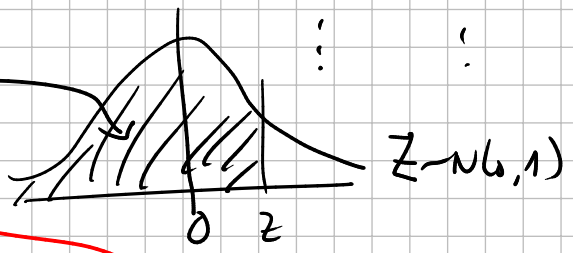
IID r.v.'s.

As $n \to \infty$

$$\mathbb{P}\left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \le z\right) \to \Phi(z)$$

$\mathbb{P}(Z \le z)$

$Z \sim N(0,1)$



$$\mathbb{P}(\bar{X}_{10} \ge 4.2) = 1 - \mathbb{P}(\bar{X}_{10} \le 4.2)$$

$$= 1 - \mathbb{P}\left(\frac{\bar{X}_{10} - 4.5}{\sqrt{1.05/10}} \le \frac{4.2 - 4.5}{\sqrt{1.05/10}}\right)$$

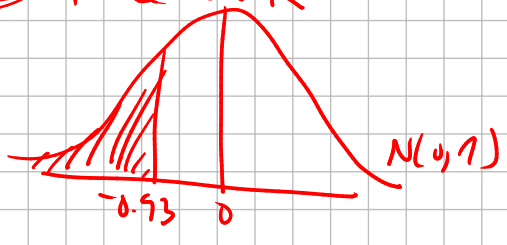
By central limit theorem

$$\approx 1 - \mathbb{P}(Z \le -0.93)$$

$$= 1 - 0.17619$$

$$= 0.82381$$

table in R



μ ↳ don't know value of μ!

$\bar{X}_n$

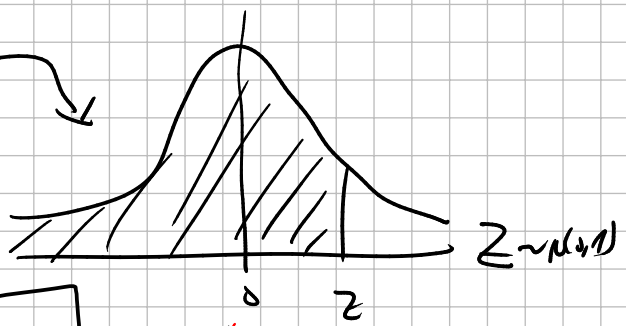
$\mathbb{P}\left(? \leq \mu \leq ? \right) = 100(1-\alpha)\%$ — confidence level

α small (near zero)

Central limit theorem

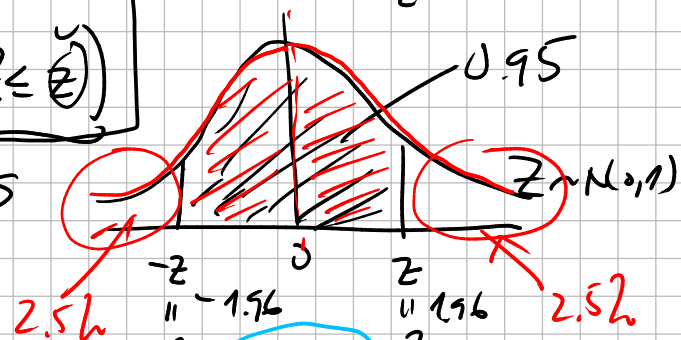
as $n \to \infty$

$\mathbb{P}\left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \leq z \right) \to \Phi(z)$, $\Phi(z) = \mathbb{P}(Z \leq z)$

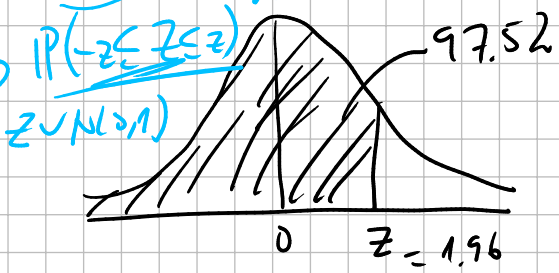


$\mathbb{P}\left(-z \leq \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \leq z \right) \to \mathbb{P}(-z \leq Z \leq z)$

0.95



$\mathbb{P}\left(\bar{X}_n - 1.96\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X}_n + 1.96\frac{\sigma}{\sqrt{n}} \right) \to \mathbb{P}(-z \leq Z \leq z) = 0.95$



- Lecture 5 d Thm 1

S_n^2 sample variance

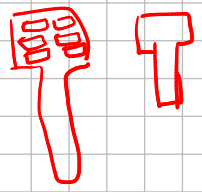
$\Rightarrow \mathbb{E}(S_n^2) = \sigma^2$

μ ?

S_n sample std dev.

? $\mathbb{E}(S_n) = \sigma$

$\mathbb{E}(S_n) \overset{?}{\geq} \sigma$



14.5 g

μ = average weight of M&M's in the bags
(package size in first rule of class)

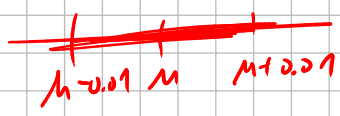
$$\mathbb{P}\left(\bar{X}_n - 1.96\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X}_n + 1.96\frac{\sigma}{\sqrt{n}}\right) \approx 0.95$$

$$\mathbb{P}\left(|\bar{X}_n - \mu| \le 1.96\frac{\sigma}{\sqrt{n}}\right) \approx 0.95$$

margin of error

$$\mathbb{P}\left(|\bar{X}_n - \mu| \le 0.01\right) \approx 0.95$$

$$1.96\frac{\sigma}{\sqrt{n}} = 0.01$$



sample size calculation